

Mathematics I — Quiz 1

Formula Sheet

Weeks 1–4 (BSMA1001) | Quiz 1: 19 July 2026

Week 1: Number Systems, Sets, Relations & Functions

Number systems: $\mathbb{N} \subset \mathbb{W} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

- $\mathbb{N} = \{1, 2, 3, \dots\}$, $\mathbb{W} = \{0, 1, 2, \dots\}$
- $\mathbb{Z} = \{\dots, -1, 0, 1, \dots\}$, $\mathbb{Q} = \{p/q : q \neq 0\}$
- Irrational = non-terminating, non-repeating decimal; $\mathbb{R} = \mathbb{Q} \cup \text{Irrationals}$

Set operations:

- $A \cup B$ (union), $A \cap B$ (intersection), $A - B$ (difference), $A' = U - A$ (complement)
- $|A \cup B| = |A| + |B| - |A \cap B|$
- $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$
- Power set: $|P(A)| = 2^n$, where $n = |A|$
- De Morgan's: $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$
- Cartesian product: $|A \times B| = |A| \cdot |B|$

Relations ($R \subseteq A \times B$):

- Reflexive: $aRa \forall a$
- Symmetric: $aRb \Rightarrow bRa$
- Transitive: $aRb, bRc \Rightarrow aRc$
- Antisymmetric: $aRb, bRa \Rightarrow a = b$
- Equivalence relation = reflexive + symmetric + transitive

Functions $f : A \rightarrow B$:

- Domain, co-domain, range ($\text{range} \subseteq \text{co-domain}$)
- One-one (injective): distinct inputs $\rightarrow$ distinct outputs
- Onto (surjective): $\text{range} = \text{co-domain}$
- Bijective = one-one + onto $\Rightarrow$ only these are invertible
- Even: $f(-x) = f(x)$ Odd: $f(-x) = -f(x)$
- Composition: $(f \circ g)(x) = f(g(x))$
- Inverse: $f(f^{-1}(x)) = x = f^{-1}(f(x))$

Week 2: Coordinate Geometry & Straight Lines

- Distance: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Midpoint: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$
- Slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \tan \theta$
- Slope-intercept form: $y = mx + c$
- Point-slope form: $y - y_1 = m(x - x_1)$
- Two-point form: $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$
- Intercept form: $\frac{x}{a} + \frac{y}{b} = 1$
- General form $Ax + By + C = 0$: slope = $-A/B$
- Parallel lines: $m_1 = m_2$ Perpendicular: $m_1 m_2 = -1$
- Angle between lines: $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$
- Point-to-line distance: $d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$

- Least-squares line fit $y = mx + c$:

$$m = \frac{n \Sigma xy - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2}, \quad c = \bar{y} - m\bar{x}$$

Week 3: Quadratic Functions

- General form: $f(x) = ax^2 + bx + c$, $a \neq 0$
- Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Discriminant $D = b^2 - 4ac$: $D > 0 \Rightarrow 2$ distinct real roots; $D = 0 \Rightarrow 1$ repeated root; $D < 0 \Rightarrow 2$ complex roots
- Sum of roots: $\alpha + \beta = -b/a$ Product: $\alpha\beta = c/a$
- Vertex form: $f(x) = a(x - h)^2 + k$, where $h = -\frac{b}{2a}$, $k = c - \frac{b^2}{4a} = f(h)$
- Vertex = $\left(-\frac{b}{2a}, -\frac{D}{4a} \right)$; axis of symmetry: $x = -b/2a$
- $a > 0 \Rightarrow$ opens up, vertex = minimum
- $a < 0 \Rightarrow$ opens down, vertex = maximum
- Factored form: $f(x) = a(x - \alpha)(x - \beta)$
- y -intercept = $(0, c)$; x -intercepts = real roots of $f(x) = 0$

Week 4: Algebra of Polynomials

- $p(x) = a_n x^n + \dots + a_1 x + a_0$, $a_n \neq 0$, degree = n
- Addition/subtraction: combine like-degree terms
- Multiplication: distribute every term over every term
- Division algorithm: $p(x) = q(x)g(x) + r(x)$, with $\deg r < \deg g$
- **Remainder Theorem:** remainder of $p(x) \div (x - a)$ is $p(a)$
- **Factor Theorem:** $(x - a)$ is a factor of $p(x) \iff p(a) = 0$
- Zero/root c : $p(c) = 0 \Rightarrow x$ -intercept at $x = c$
- Multiplicity m of root c (power of $(x - c)$ dividing p): odd $m \Rightarrow$ graph crosses axis; even $m \Rightarrow$ graph touches & turns back
- Degree n polynomial: at most n real zeros; at most $(n - 1)$ turning points
- End behaviour (from leading term $a_n x^n$):
 - n even, $a_n > 0$: both ends $\uparrow$ n even, $a_n < 0$: both ends $\downarrow$
 - n odd, $a_n > 0$: left $\downarrow$, right $\uparrow$ n odd, $a_n < 0$: left $\uparrow$, right $\downarrow$

Statistics 1 — Formula Sheet (Weeks 1-4)

ITM BS Foundation | Quiz 1 Revision — 19 July

Week 1 — Basics of Data

- **Population** (entire group) vs **Sample** (subset studied)
- **Case** (unit/subject) vs **Variable** (characteristic measured)
- **Descriptive statistics** (summarize data) vs **Inferential statistics** (generalize sample → population)
- **Data types**: Qualitative (categorical) | Quantitative → Discrete / Continuous
- **Scales of measurement**: Nominal → Ordinal → Interval → Ratio (increasing information content)
- **Sampling methods**: Simple random, Stratified, Systematic, Cluster, Convenience

Week 2 — Organizing & Visualizing Data

- **Relative frequency** = $f \div n$
- **Cumulative frequency** = running total of frequencies
- **Class width** = $(\text{Max} - \text{Min}) \div \text{number of classes (round up)}$
- **Class midpoint** = $(\text{Lower limit} + \text{Upper limit}) \div 2$
- **Graphs**: Bar chart, Pie chart, Pareto chart (categorical) · Histogram, Stem-and-leaf plot (numerical)
- **Shape of distribution**: Mean > Median → right-skewed | Mean < Median → left-skewed | Mean $\approx$ Median → symmetric

Week 3 — Central Tendency & Position

- **Mean**
 - Population: $\mu = \Sigma x \div N$
 - Sample: $\bar{x} = \Sigma x \div n$
 - Grouped data: $\bar{x} = \Sigma(f \cdot m) \div \Sigma f$ (m = class midpoint)
- **Median** — middle value of data sorted ascending (average of two middle values if n is even)
- **Mode** — most frequently occurring value
- **Percentile (pth) — Weiss method** (used in this course):
 - 1. Compute $i = (p \div 100) \times n$
 - 2. i not a whole number → round UP → value at that position = pth percentile
 - 3. i IS a whole number → average the values at position i and $i+1$
- **Q1** = 25th percentile · **Median (Q2)** = 50th percentile · **Q3** = 75th percentile
- **Z-score**: $z = (x - \mu) \div \sigma$ (sample: $z = (x - \bar{x}) \div s$)

Week 4 — Dispersion & Summary Measures

- **Range** = Max – Min
- **Population variance**: $\sigma^2 = \Sigma(x - \mu)^2 \div N$
- **Sample variance**: $s^2 = \Sigma(x - \bar{x})^2 \div (n - 1)$
- **Grouped variance**: $s^2 = \Sigma f(m - \bar{x})^2 \div (n - 1)$
- **Standard deviation**: $\sigma = \sqrt{\sigma^2}$ | $s = \sqrt{s^2}$
- **Range rule of thumb**: $SD \approx \text{Range} \div 4$ (quick estimate)
- **Coefficient of Variation**: $CV = (s \div \bar{x}) \times 100\%$
- **IQR** = $Q3 - Q1$
- **Five-Number Summary**: Min, Q1, Median, Q3, Max
- **Outlier rule**: value is an outlier if below $(Q1 - 1.5 \times IQR)$ or above $(Q3 + 1.5 \times IQR)$
- **Empirical Rule** (bell-shaped data only): 68% within $\mu \pm 1\sigma$, 95% within $\mu \pm 2\sigma$, 99.7% within $\mu \pm 3\sigma$
- **Chebyshev's Theorem** (any distribution shape): at least $(1 - 1/k^2)$ of data lies within k standard deviations of the mean ($k > 1$)